How well can HMM model load signals

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How to model loads

Hidden Markov Models (HMM)

- Basics
- HMM states vs. load states
- Restricting the HMM parameters

Model selection

- Akaike Information Criterion (AIC)

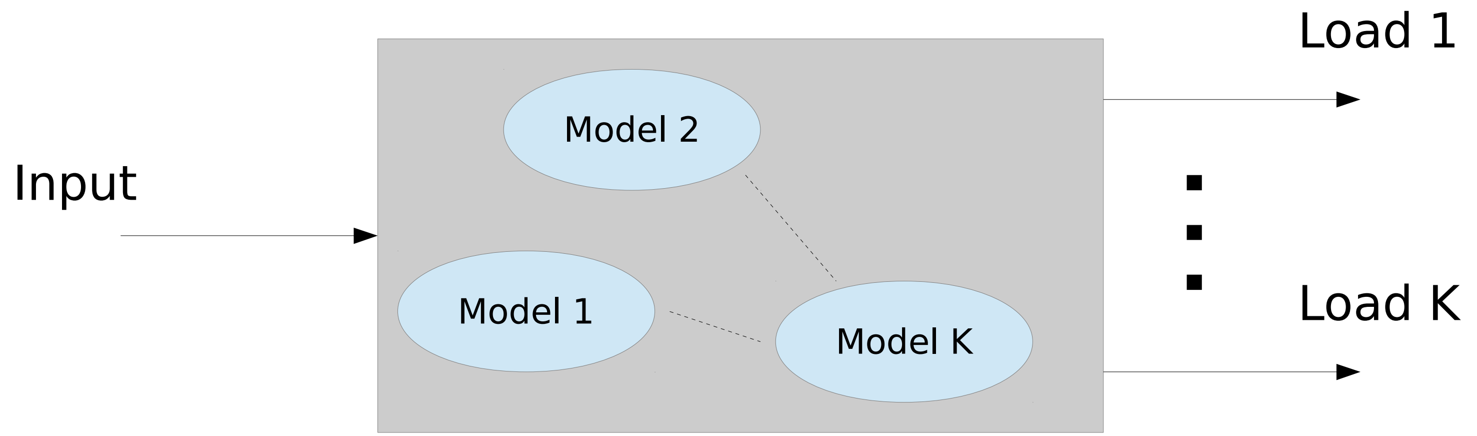
Model adaptation

- An easy parametric transformation

Experiments

How to model loads

Non-Intrusive load monitoring as a single channel source separation problem



Data acquisition

- Single channel
- Low frequency
- Real power only
- Hierarchical time dependencies
- Non-stationary

Separation

- Factorization/clustering methods
- Methods for denoising
- Bayesian methods

Main problem

- We need suited signal models to perform separation

How to model loads

State of the art

Piecewise modelling

- Event based approaches
- Problem of segmentation
- Loss of information for variable loads
- Captures little information about time dependencies

Recurrent Neural Network

- Very powerful model
- Can learn hierarchical time dependencies
- Hard to train
- Hard to interprete

Hidden Markov Model

- Simple model that can capture dependencies between adjacent states
- Easy to train
- Good results if used with fHMM
- No investigation yet how well they fit to load signals
- Good to interprete?

Questions related to HMM

Goodness of fit

- Are HMMs suited to model all kind of loads?
- How can we interprete the HMM states? Are they equal to the physical load states?
- How to choose the number of states?

Model adaptation

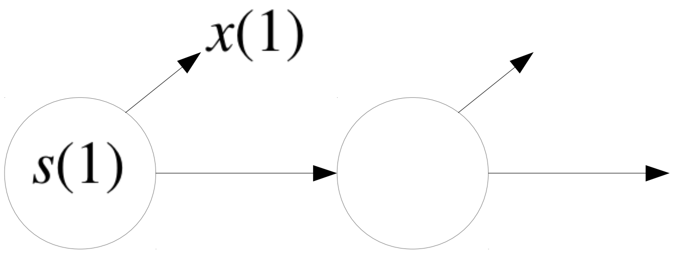
- Can we adapt HMMs to other houses?

Hidden Markov Models

Basics

Observation sequences and state sequences

$$X = \{x(1), \ldots, x(N)\} \qquad x(n) \in \mathbb{R}$$
$$S = \{s(1), \ldots, s(N)\} \qquad s(n) \in \{1, \ldots, M\}$$



Initial state and state transition probabilities

$$\pi_i = P(s(1) = i), \; 1 \leq i \leq M, \; \underline{\pi} = [\pi_1, \ldots, \pi_M]^T \in \mathbb{R}^M$$

$$a_{ij} = P(s(n) = i | s(n-1) = j), \; 1 \leq i, j \leq M$$
$$\mathbf{A} = [a_{ij}] \in \mathbb{R}^{M \times M}$$

Emission probabilities

$$\rho_i(x(n)) = p(x(n) | s(n) = i), \; 1 \leq i \leq M$$

$$\rho_i(x(n)) \sim N(\mu_i, \sigma_i^2)$$

Hidden Markov Models

Basics

Joint sequence probability

$$p(X, S\,|\underline{\theta}) = \pi_{s(1)}\rho_{s(1)}(x(1)) \prod_{n=2}^{N} a_{s(n)s(n-1)}\rho_{s(n)}(x(n))$$

Training and state inference

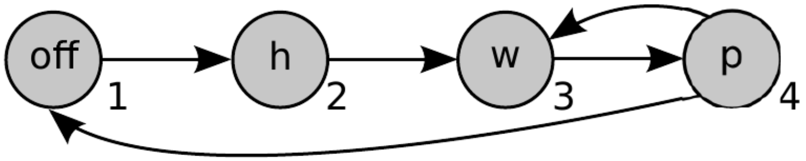
$$\hat{\underline{\theta}} = \arg\max_{\underline{\theta}} \prod_{k} p(X_k|\underline{\theta})$$ Baum-Welch algorithm

$$\hat{S} = \arg\max_{S} p(X, S\,|\hat{\underline{\theta}})$$ Viterbi algorithm

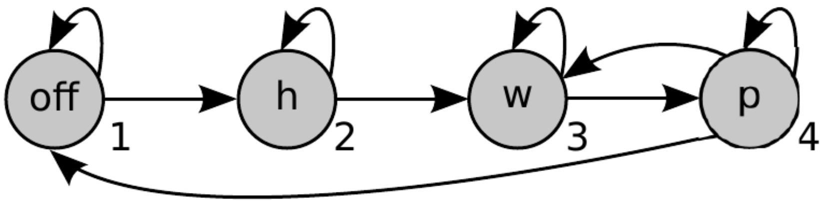
Hidden Markov Models

HMM states vs load states

Transition of load states



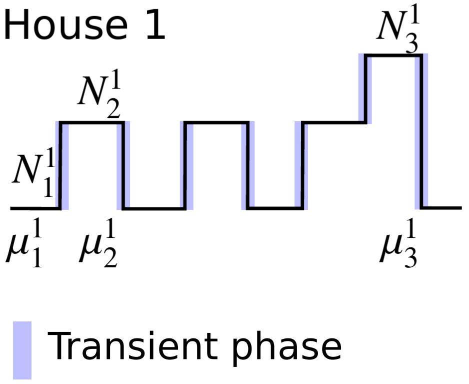
Transition of HMM states



- In general the load states and HMM states are different
- Their relationship depends on
  - Type of the load
  - Sampling frequency
  - Transient phase of the load

- HMM states = load states if
  - States with perfectly constant power consumption
  - Sharp transient phase

House 1



Hidden Markov Models

Controlled vs uncontrolled

controlled multi-state loads	uncontrolled multi-state loads	variable loads
washing machine dishwasher microwave toaster	fridge lighting ciruit	PC amplifier

Restricting the HMM parameters

- For some controlled loads we can use prior knowledge to reduce the number of HMM parameters
  → better estimate of parameters

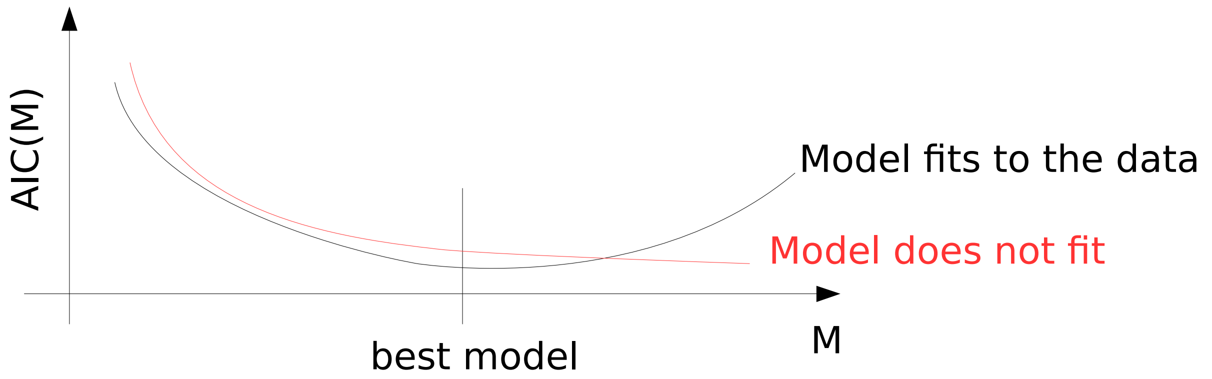
- Example: periodic chain structure leads to special transition matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & 0 & 0 & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{pmatrix}, \quad a_{ij} \neq 0.$$

Model selection

Akaike Information Criterion (AIC)

$$AIC(M) = -2L_M + 2N_M, \;\; L_M = \ln p(X|\underline{\hat{\theta}}_M)$$

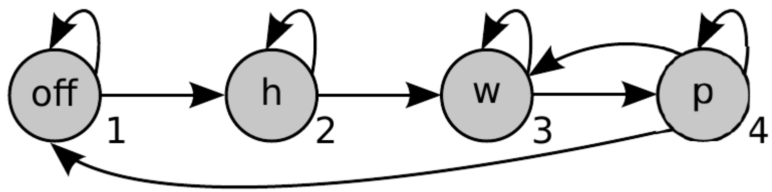


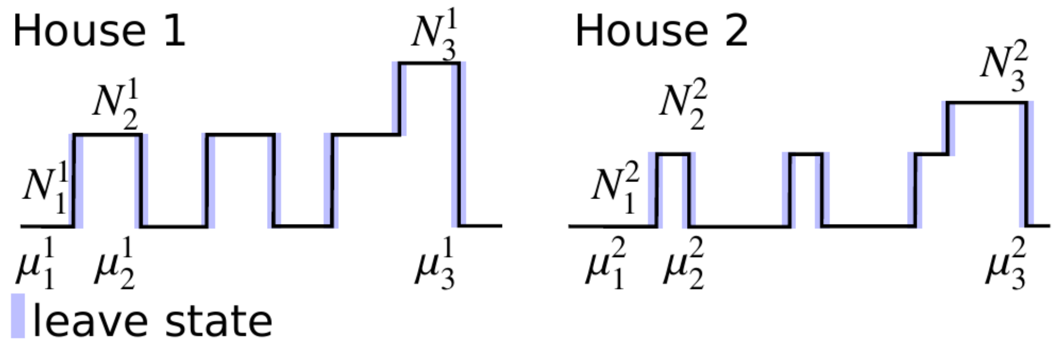
- Measure how well a model fits to a specific load
- Balances goodness of fit (data likelihood) against model complexity (number of parameters M)
- Choose model with lowest AIC

- If model does not fit
  - Increasing model complexity always leads to increasing data likelihood

Model adaptation

Basics





What are causes for differences between signals of loads of the same kind

- Differences in sampling frequency
- Differenent power consumption in each state
- Different state duration

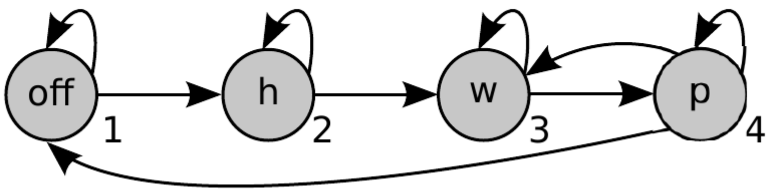
Assumptions

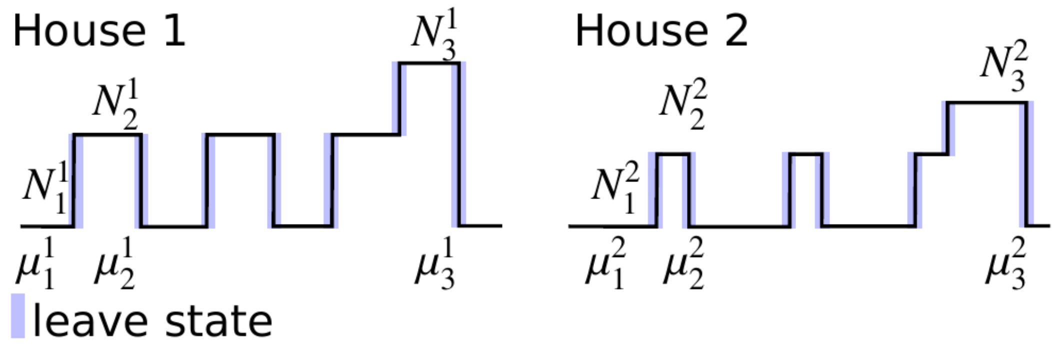
- Periodic signal patterns
- Loads of the same kind share the same set of states
  → We can use a simple transformation of parameters to adapt the HMM

$$\underline{\hat{\theta}}_2 = f(\underline{\hat{\theta}}_1, \underline{\alpha})$$

Model adaptation

Adaptation of the state mean





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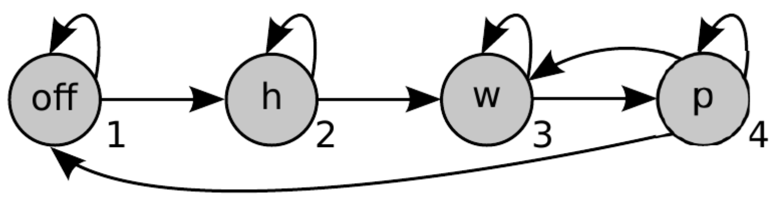
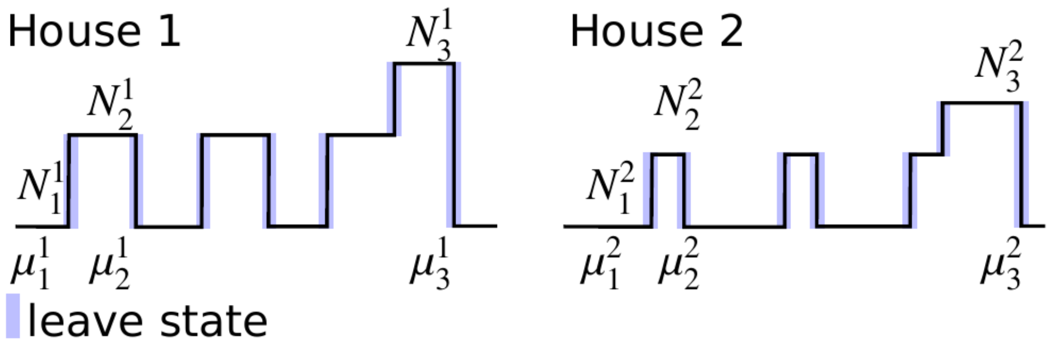
→ The means of each emission probability of all states are scaled independently

$$\mu_i^2 = \alpha_i \mu_i^1, \quad 1 \le i \le M.$$

Model adaptation

Adaptation of the transition matrix





What are causes for differences between signals of loads of the same kind

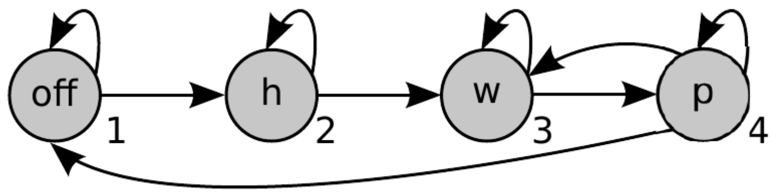
- Differences in sampling frequency
- Differenent power consumption in each state
- Different state duration

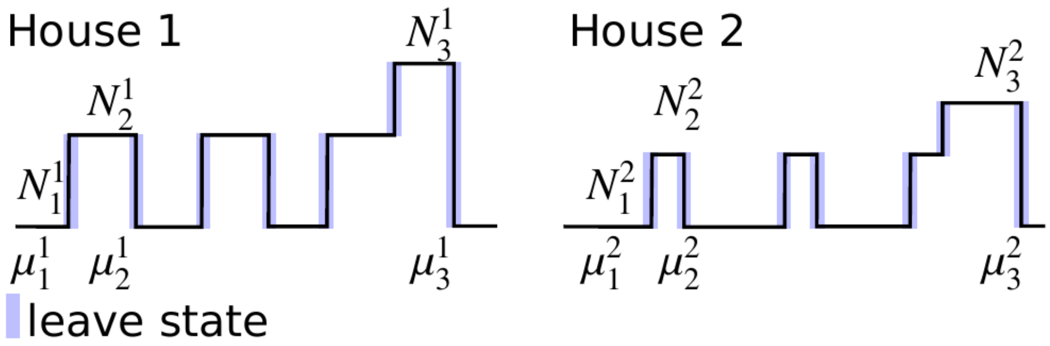
$$N_i^1 \rightarrow N_i^2 = c_i N_i^1$$
$$\mathbf{A}^1 \rightarrow \mathbf{A}^2$$

$$
\begin{aligned}
a_{ii}^k &= P^k(s(n) = i|s(n-1) = i) = \frac{N_i^k - 1}{N_i^k},\ 1 \leq i \leq M \\
a_{ij}^k &= P^k(s(n) = i|s(n-1) = j) \\
&= P^k(s(n) = i|l_j(n))\, \underbrace{P^k(s(n) \neq j|s(n-1) = j)}_{1-a_{jj}^k},\ i \neq j
\end{aligned}
$$

Model adaptation

Adaptation of the transition matrix





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$$N_i^1 \rightarrow N_i^2 = c_i N_i^1$$
$$\mathbf{A}^1 \rightarrow \mathbf{A}^2$$

Re-scaling the diagonal elements

$$a_{ii}^2 = \frac{N_i^2 - 1}{N_i^2} = \underbrace{\frac{c_i N_i^1 - 1}{c_i (N_i^1 - 1)}}_{\lambda_i} a_{ii}^1, \quad 1 \leq i \leq M$$

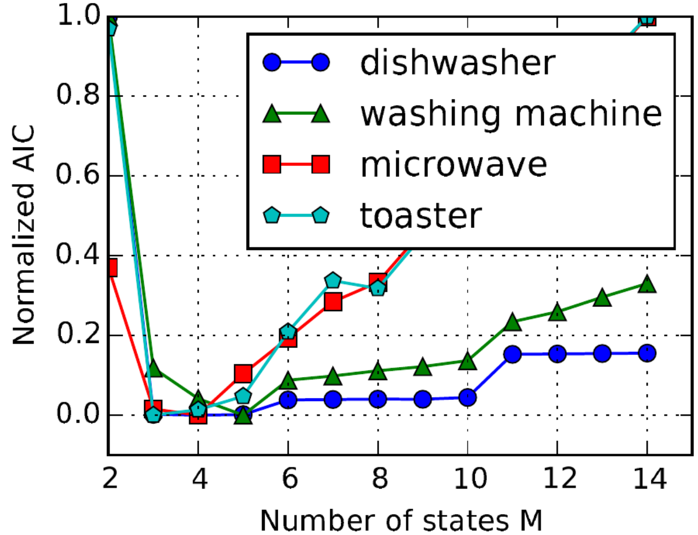
Re-scaling the off-diagonal elements

$$\begin{aligned} a_{ij}^1 &= P^2(s(n) = i | l_j(n))(1 - a_{jj}^2) \\ &= P^1(s(n) = i | l_j(n))(1 - a_{jj}^2)\frac{1 - a_{jj}^2}{1 - a_{jj}^1} \\ &= \frac{1 - \lambda_i a_{11}^1}{1 - a_{ii}^1}, \quad i \neq j \end{aligned}$$

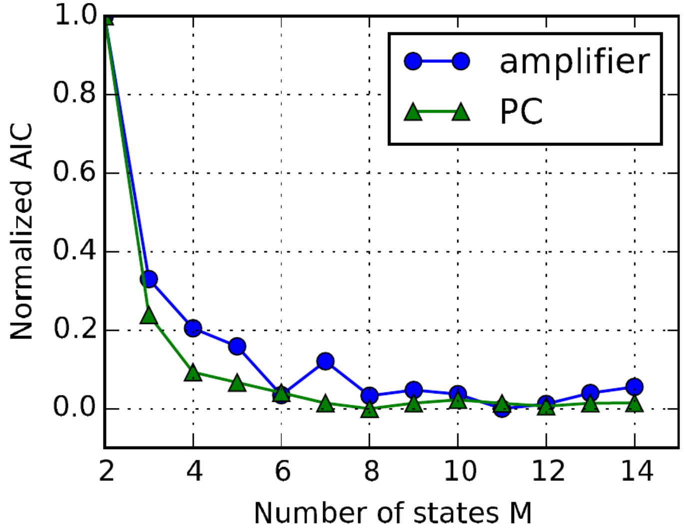
Experiments

AIC for different loads

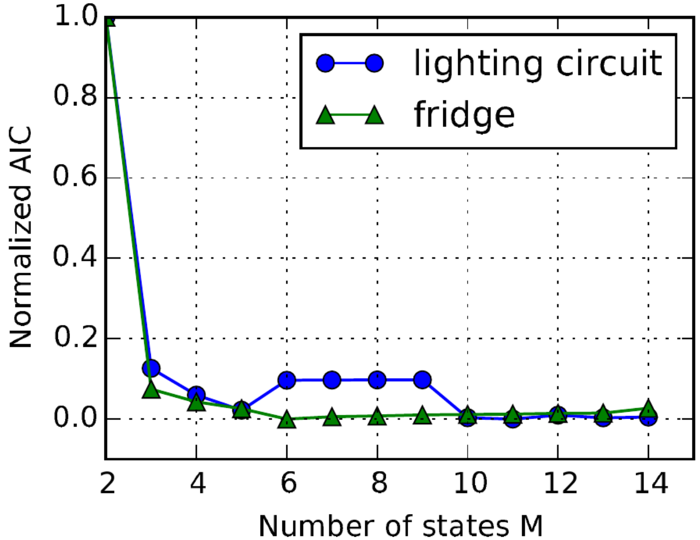
Controlled multi-state load



Variable load



Uncontrolled multi-state load

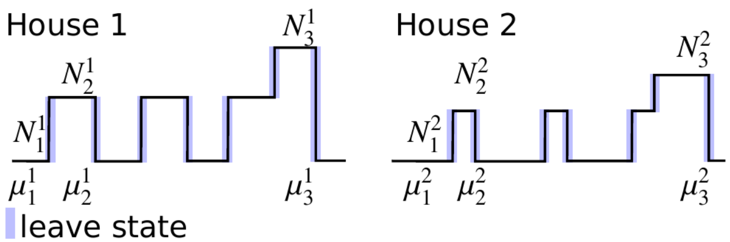


Result

- Clear minima for controlled multi-state load using only few HMM states
- AIC keeps decreasing for increasing number of states (increasing model complexity) in case of uncontrolled multi-state and variable loads

Experiments

Adaptation to simulated data



	μ_1^2	μ_2^2	μ_3^2	a_{11}^2	a_{22}^2	a_{33}^2
true	0	1.2	2.3	0.875	0.95	0.955
estimated	0	1.19	2.3	0.875	0.94	0.97
	μ_1^1	μ_2^1	μ_3^1	a_{11}^1	a_{22}^1	a_{33}^1
orig.	0	1	2	0.8	0.9	0.95

- Simulated periodic load signal
- Train on 350 samples of house 1 (10 periods)
- Adapt on 35 samples of house 2 (1 period)

Adaptation to measured data

load	$\ln p(X_1 \vert \underline{\hat{\theta}}_1)$	$\ln p(X_2 \vert \underline{\hat{\theta}}_1)$	$\ln p(X_2 \vert f(\underline{\hat{\theta}}_1, \underline{\alpha}))$
fridge	3311	1430	3005
dishwasher	2881	-1614	2453
washing machine	7100	2369	3375

Conclusion

Is HMM a suited model for all loads?

- Good model for controlled multi-state loads with fixed periodic behaviour
- Bad model for uncontrolled multi-state and variable loads

Can we adapt HMM to different houses?

- For periodic signals that share the same set of states between houses
  → parametric transformation of model parameters
  can be used for adaptation
- Only little data for adaptation is needed

Outlook

- For which loads can we reduce the model complexity by restricting the model parameters?
- How do we have to modify the HMM assumptions to get good models for variable and uncontrolled multi-state loads?